Expert Defined versus Learned Dynamic Bayesian Networks for Inflation Prediction

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1 Introduction

Inflation is the rise and fall in the purchasing power of money. It is a key macroeconomic variable in the US economy and control over it is often sought by the Federal Reserve, which can raise and lower the Federal Reserve rate in an attempt to control it. Other macroeconomic variables are at play, however, and much research has been done to create predictive models [1] [6]. Such models are often defined by experts in the field and are based on extensive domain knowledge. One proposed macroeconomic model of inflation is the "simple macroeconomic model" (SMM) [6]. The explicit equations of this model parameterize inflation with four variables: the federal funds rate, the GDP, labor productivity, and wages.

However, the use of graphical models has also begun to take hold in economics [2] [4]. Such models allow for better understanding of causal relations and mechanisms that determine the workings of the economy. Dynamic Bayesian networks in particular have shown much promise in exploiting correlations to make predictions about macroeconomic trends, a form of inference [4]. However, these models also allow us to learn the structure of the system we are modeling without extensive domain knowledge, as is done in the SMM model. We propose the use of structure learning to create a better model of inflation given the options of the same variables used in the SMM model. We will test our model through dynamic Bayesian network inference, specifically predictions of future levels of inflation. We will conduct inference in the setting of the SMM model structure as well as a benchmark level of performance and to compare how an expert-defined structure compares to a data-driven structure.

2 Data

The data for inflation and each of the four other variables come from the publicly available economic database FRED, run by the Economic Research Federal Reserve Bank of St. Louis [5]. Data for all five variables were obtained from 1980 through 2019 at quarterly intervals. The variables obtained, specifically, were *Consumer Price Index for All Urban Consumers: All Items Less Food and Energy*, *Effective Federal Funds Rate*, *Employed full time: Median usual weekly real earnings: Wage and salary works: 16 years and over*, *Nonfarm Business Sector: Real Output Per Hour of All Person*, and *Real Gross Domestic Product*. Each variable was expressed as a percent change from a year ago, except for the federal funds rate which was a raw percent, as set by the Federal Reserve. Data which was sampled more frequently than quarterly was averaged. The consumer price index, CPI, is the measure upon which inflation is estimated. Thus, it's percent change from a year ago represents the current inflation rate.

3 Methods

A dynamic Bayesian network is akin to a vector auto-regressive (VAR) process in its modeling of dependency relations on past time states. Many models, including the SMM [6], often assume dependencies on just the prior time state and thus are order 1 models. An order 1 VAR process is defined as

$$X(t) = AX(t-1) + B + \epsilon(t)$$

where $epsilon(t)$ is a normally distributed random variable, independent of the other states and time invariant. Values in the A matrix define linearly dependent relations, with nonzero values translating to edges in a dynamic Bayesian representation.

A further assumption in this case is that the network itself is homogenous, that is that the parameters and structure do not change in time. Learning of the structure of this homogeneous Bayesian network requires adding and/or removing arcs in the network using some algorithmic process. Structural learning in this case will be limited to an order one process, as the SMM model is defined on, except in the case of the Federal Funds Rate. Based on examination of the partial autocorrelation function, significant lags were observed between the Federal Funds rate and the interest rate up until lag four.

3.1 Structure Learning

The first structural learning method applied is the *Least Absolute Shrinkage and Selection Operator*, better known as LASSO [7]. This is constraint-based learning, where edges are identified by significant results of conditional independence tests. CPI is our target variable to predict and observation matrix of variables at the prior time step serves as the potential set of parents. An L_1 penalty constrains the sum of the absolute regression coefficients and the lasso solution is computed for all fractions of the L_1 norm, from 0 to 1. Cross-validation yields the lasso solution with the minimum mean-squared error which is then used to compute the final set of edges.

A second structural learning algorithm is G1DBN [3], a two-step score-based algorithm which considers candidate DAGs and maximizes a score objective function over them and then infers the underlying dynamic Bayesian network. In the first step, first-order dependencies are encoded as a scored and thresholded DAG. Then, subgraphs are considered as potential dynamic Bayesian networks and an α value of 0.05 yields a statistically significantly edge set.

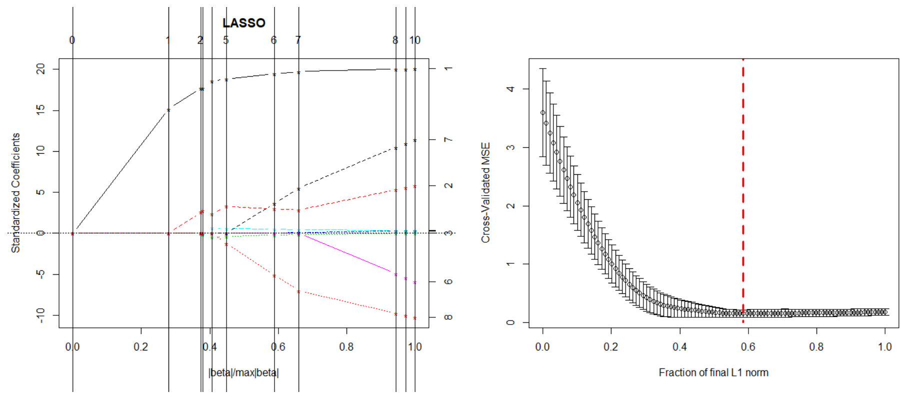
3.2 Inference

For each model, one-step predictions were made over the course of the latter 24 quarters, representing the testing data set. All prior quarters composed the testing data set. Each model specified the set of variables upon which CPI was ruled to be conditionally dependent on and these models were fit to the training set by a maximum likelihood method from the *bnlearn* package.

Dynamic Bayesian networks allow for three main forms of inference: smoothing, filtering, and prediction. Here, we are interested in prediction as that allows us to estimate the future values of inflation. Using all available nodes as evidence, observations were sampled from likelihood weighted simulations. The expected value of the simulation was taken to be the predicted value. For each of the three structural models, the predictions were made and their performances were gauged by examining the root mean squared errors.

4 Results

The LASSO graph was optimally fit with a BIC score of -1945 using an L_1 penalty coefficient to maximize the mean-squared error as depicted in Figure 1. Inflation was found to be conditionally dependent on inflation, the Federal Funds rate, wages, and GDP at lag one as well as the third and especially fourth Federal Fund rate lags. Notably, inflation was not found to be dependent on the total labor output, as was proposed by the SMM model. The G1DBN learned structure was fit with an optimal BIC score of -706, much lower than the LASSO structure. However, it found inflation to be only dependent on inflation and the federal funds rate at lag one, a very simple model and defying most of what the SMM proposed. Lastly, the SMM dependency structure was fit with a BIC of -1479. All three models were subsequently tested on future time points, as shown in 2, and the RMSEs are shown in 1.



(a) (b)

Figure 1: Results of the **LARs** package. *Left:* Learning of the edges to include. *Right:* Cross-validation estimates. The vertical line denotes the L_1 norm penalty at the minimum MSE.

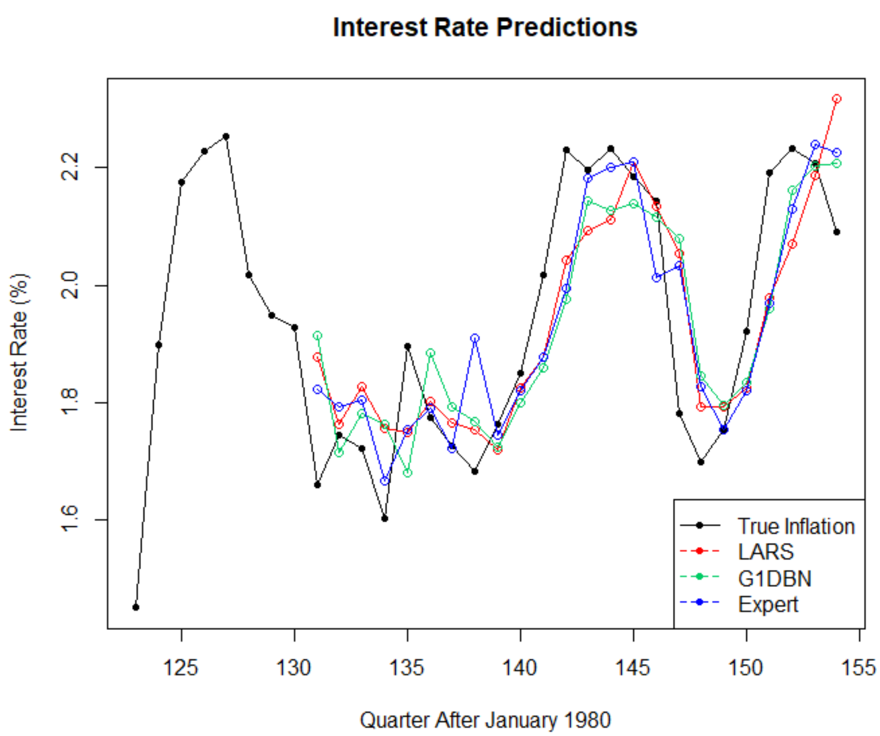


Figure 2: Prediction results comparing the two learned models and the expert specified model.

	LARs	G1DBN	Expert Model
RMSE	0.131	0.139	0.125

Table 1: Root-mean-square errors of the 24 quarter predictions for each model structure

5 Discussion

Although all three root-mean-squared error results are similar, the expert-defined model has the lowest error, with the LARs derived structure coming in second. Based on the prediction plots, all three seemed to be decently well fitted so as to make accurate predictions. Interestingly, the G1DBN model did the poorest of all and yet had the best BIC score during testing. It appears that this is due to its fitted model having very few edges and thus the penalty score term was minimized. Yet this failed to give the model enough predictive power. Using the AIC may have resulted in better end performance since many of the relations were disregarded on the basis conditional independence. At a higher level, we would hope to add further macroeconomic variables to our structural learning procedures as that would potentially allow for more accurate models to be learned

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